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COMPARISON OF TWO MODELS TO SCHEDULE THE OPERATING THEATRE

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ABSTRACT: *The daily scheduling of the operating theatre is a highly constrained problem. Consequently, it is hard to find an optimal solution, or at least high quality solutions, in reasonable time. Through the various human and material resources, we evaluate two models that take into account a maximum of constraints encountered in real life problems. The performance of these two models are compared to determine which is the best to add a maximum of these constraints when considering the descriptive power of the two underlying paradigms, which are the mixed-integer and constraint programming.*

KEYWORDS: *models comparison, operating theatre, constraint programming.*

1. INTRODUCTION

The health systems of many countries are in crisis. This situation is not new but in contrast, what is new is the sudden awareness that if the current trend continues, most health systems will be not more no longer viable in 2015 (IBM, 2006).

Belgium does not make exception, even if it has, as generally suggested, the best healthcare system in the world. Its strengths are the coverage of the population close to 100% (Durant, 2006), dynamics induced by the global supply system and the actual delivery of care, basic education and quality healthcare providers and the increasing professionalization of management structures of healthcare (Itinera Institute, 2008). Nevertheless, Belgium cedes ground to other countries as regards the overall quality of healthcare systems. Belgium occupies the eleventh place ranking in 2009 of *Euro Health Consumer Index* (EHCI, 2009). The first place ranking is occupied by the Netherlands, followed by Denmark, Iceland, Austria, Switzerland (Belga, 2008). But healthcare systems are fundamental in our society and are the engines of human activity and therefore economic development. The health and its preservation are human needs that everyone wants to obtain and maintain. This is why society has the means to facilitate access to healthcare (both in terms of offerings and financial aspects) (AGIM, 2009). The quality and accessibility of the Belgian healthcare system are historically based on three fundamental bases: (i) the medical and paramedical staff is highly qualified, (ii) public or collective funding is generous and (iii) the market is open to healthy competition between physicians, hospitals and health insurance funds (Itinera Institute, 2009)

During the sixties and seventies, the prosperous economic situation allowed expenses without constraints, which lead to hire enough people and buy enough material to ensure good quality services.
During the eighties then, and up to 2007, three main rationalization axes appeared which focused on the hospitals, on the complementary health insurances and on the Inami[1], and then on the pharmaceutical industries, on the doctors and on the health coverage.

Besides, the part taken by healthcare expenses in public funds has kept growing continuously over the last years; and this growth is faster than the GDP[2] evolution (FEB, 2003). This has been confirmed over the last few years, with an annual growth close to 5% (inflation included). But besides public funds, the number of private insurances has grown exponentially. Almost one third of the total Belgian healthcare costs is already financed through private funds (Itinera Institute, 2009).

Today, the Belgian State takes rationalization measures to counteract the constant growth taken by the healthcare budget in welfare expenses. These measures affect directly the hospital system.
This research focuses on the operating theatre management because it takes a central position in the hospital activity and represents one of its most expensive sectors. The operating theatre in a hospital is composed of operating rooms and one recovery room. This research aims to help the operating theatre manager in order to improve its organization, in particular the surgical cases assignment. This kind of problems aims at assigning a set of

[1] Institut national d'assurance maladie invalidité
[2] Gross Domestic Product

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surgical cases to operating rooms during one period (often one week). This weekly operating theatre planning and scheduling problem is solved in two phases. First, a planning problem is solved to give the date of surgery for each patient, allowing for the availability of operating rooms and surgeons. Then a daily scheduling problem is solved to determine, each day, the sequence of operations in each operating room. (Fei *et al*., 2009)

The operating theatre management is a complex task because surgical cases must be planned and scheduled in order to minimize the costs of operating rooms and also satisfying the needs and requests of the surgeons, anesthetists, nurses and patients. Moreover satisfying the patient's needs and managing all material resources also need to be taken into account. Besides the human and material resources are in limited quantity, legal regulations have to be respected, etc.

We focus our study on the daily surgical cases scheduling taking into account human and material constraints. We consider the widely used scheduling strategy in Belgian hospitals: the block scheduling, where surgeons are affected to a time slot to operate their patients.

Efficiently scheduling the surgical cases is essential to a punctual implementation of the operative programming, leading to no delay. Delays are not only a source of stress for the teams, which are trying to perform good quality work, but they also represent a significant cost, through the overtime hours generated (Perdomo *et al.*, 2006). Indeed, overtime hours are more costly than regular hours, and they must also be recuperated by the nursing staff. Moreover, considering that the nursing staff is not regularly present, it leads to additional complications of the conception and implementation of a weekly schedule. The operative program has therefore a direct impact on the quality of care given to the patients, and on their perception towards the hospital.

The place taken by the human being in the decision process cannot be ignored. The operating theatre gives rate to various teams (surgical, nursing, anesthetists, maintenance…). Considering the above-mentioned limitations on resources, the schedule of their activities is crucial to an efficient implementation of the joint work.

Each resource limitation included in an operating theatre management model leads to an increase of its complexity (in terms of the number of variables and/or the number of constraints and the computational time required to solve the resulting optimization problem). Some constraints related to the resources are linked to the fact that these resources are only available in limited number or in limited capacity: opening hours of the operating rooms, availability of the surgeons, anesthetists, nurses, availability and number of surgical equipments, availability and number of recovery beds. Other constraints are linked to the competence of the resources such as the versatility of the operating rooms, the staff qualification. In the same way, as the operating rooms may be dedicated to certain types of surgical procedures, medical staff may also be specialized. The operating theatre manager tries therefore to harmonize resources and provide more versatility to his staff to optimize the operating theatre.

In this paper, the aim of the "optimization" phase of the problem is to provide the operating theatre with the best conditions of use, functioning and efficiency. This optimization leads to a better use of the material and human resources, guided by a specific objective materialized by a cost function. The objective of the optimization is to create an "optimal" planning and/or schedule, in the sense that it fits the best ideal values fixed for a set of predefined performance measures.

The optimization of the operating theatre is mainly translated in terms of costs (rooms opening costs, running costs, staff and material costs, overtime hours costs…) (Macario, 2007; Lamiri *et al.* 2008; Hans *et al.* 2008; Denton *et al.* 2007) or in terms of an objective function which includes various performance indicators such as waiting time, intermediate time, delays, overtime hours etc. (Macario, 2007). The time indicators can often be translated in terms of costs if the expenses occurring during the induced time slots can be identified. This is why the overtime hours and associated costs are often taken into account in the performance indicators. The omnipresent objective is undoubtedly the improvement of the care quality (including the patients security and satisfaction) (Testi *et al.* 2009; Van Oostrum *et al.* 2008).

The daily operating rooms scheduling is a highly constrained problem. It is also hard to find an optimal solution or at least high quality solutions.

According to a recent review made by Hanset *et al.* (2008) about operating theatre scheduling and planning problems, we came to the conclusion that only few authors have taken into account the constraints linked to both material and human resources (Christian *et al.* 2006; Kharraja *et al.* 2004; Lamiri, 2007; Magerlain and Martin 1978). Concerning human resources, the only considered constraints are the number and availability of surgeons, stretcher-bearers and anesthetists. Generally material resources constraints are limited to the number of operating rooms and the number of beds in the recovery room.

In real-life problems, other constraints are also important to be taken into account as, for examples, human constraints like availability and preferences of surgeons, nurses and anesthetists as well as material constraints like versatility of operating rooms, availability of rooms and medical material, etc.

In this paper, we compare our scheduling model (Hanset *et al.*, 2010) to a model previously published (Roland *et al.*, 2009) that was adapted for the need of this study. The model developed by Roland *et al.* was chosen because it is one of the most extensive models, in terms of constraints, that the authors have encountered so far. The schedule obtained with this model has been solved with

a mathematical programming approach (Roland *et al.*, 2009). This model is applied to solve a scheduling problem in the class of « open-scheduling » problems, over one week.
In order to compare the two models the "Roland *et al.*" model has been adapted so that:

- it can take into account both the operating rooms and the recovery room;
- it is applied on a period of one day instead of one week;
- resources are embedded;
- a block scheduling strategy is implemented instead of an open scheduling strategy.

The "modified version" of this model, so called "first model" in the following sections will be compared with our model, so called "second model" in the following sections.

The design of our model is completely different: we developed a "generic" adaptive and modular model which embeds most of the characteristics of the problem encountered in the literature as well as real-life constraints coming from practical problems in hospitals. This second model is modular in the sense that it is composed of several independent sets of easily identifiable constraints that can be added or removed according to the elements that compose the real-life target problem.

To build this model, the constraint programming was used to express all the constraints. This allowed to include original constraints such as the priority of some surgical cases (for example people suffering from diabetes must be scheduled at the beginning of the day), availability and preferences of the staff, constraints that we have never encountered in other scheduling models but that are indeed present in real-life problems.
It is obvious that the more the constraints, the more the problem will be complex to model and to solve (Krzysztof, 2003). However, thanks to its modularity, we developed a highly adaptable tool perfectly suited to the different usual problems encountered in hospitals. The resulting model is an instantiation of the above-mentioned modular model which embeds the minimum required constraints and variables dedicated to the target hospital.

The major difficulty encountered while solving a combinatorial optimization problem is the combinatory explosion. The main problem while searching for an optimum amongst the huge set of solutions is the time taken by "this quest" of a solution expressed in terms of computing hours or days, which is not acceptable in this kind of limited-time decision making problems. However, contrary to classical mathematical approach, the constraint programming paradigm is based on "reasoning" on the constraints and is still efficient when considering a lot of constraints. This is one of the reasons why the constraint programming was chosen. This method has also the advantage of providing the user with different modes of functioning when searching for a solution in the path tree of solutions.

Comparison between the two models is thus the aim of this paper.
After this introduction, the considered problem is defined in the second part of the paper, with its notations and hypothesis; the third and fourth parts describe the two scheduling models, and the fifth part compares the two models. Finally conclusions and perspectives are presented.

2. PROBLEM STATEMENT

In order to compare our constraint programming model with a mixed integer programming model (Roland *et al.*, 2009), the following paragraphs describe the tackled problem.

We consider the daily scheduling problem of a fixed number of surgical cases. The time horizon is set to one day and discretized into T time slots (t = 1, ..., T). In our implementation each time slot is fixed to 15 minutes. This temporal granularity is quite close to reality without increasing considerably the size of the search space. There is a set of R operating rooms. Each room will then be available for T intervals of time, which will correspond to free periods for the surgeons.

In practice, there are two types of operations, the so-called elective operations, which are planned by the surgeon in consultation with the patient and the emergency operations, which – as suggested by their name – are not foreseen and arrive unexpectedly. Here, we shall be interested only in the elective operations. In our model, each operation included in the set of operations to be assigned is denoted by $o = 1, \ldots, O$. Because of medical reasons the pre-emption is not allowed (once begun an operation cannot be interrupted).

Our first model differs from the original published version of the model published by Roland *et al.* (2009) in terms of taking into account additional constraints. Firstly, we consider the existence of high and low priorities of operations. We distinguish two types of priorities. The first one allows the operation to be carried out earlier during the day; it mainly concerns children or one-day operations (out-patient, leaving hospital the same day). The second one forces operations to be carried out at the end of the day; it mainly concerns infectious cases that contaminate the room and require a particular cleaning after the operation. These two kinds of operations are materialized by two separate sets, Ω_b and Ω_e in the model. Secondly, we have added preferential constraints relative to the teamwork and to the availability of human resources. Furthermore, we introduce constraints concerning the second stage of the surgical unit, the recovery room, in the two models.

A set of notations is used to formalize both models.

2.1 Problem statement

Here are the notations used for our two models.

Ω	:	*The set of all the surgical cases.*
$\boldsymbol{O}$	:	*The number of operations.* $O = \vert \Omega \vert$
$\mathbf{o}$	:	*An operation.* $o \in \{1..O\}$
$\boldsymbol{T}$	:	*The number of time slots in a day.*
$\mathbf{t}$	:	*A time slot.* $t \in \{1..T\}$
$\boldsymbol{\Gamma}$	:	*The set of operating rooms*
$\boldsymbol{R}$	:	*The number of operating rooms.* $R = \vert \Gamma \vert$
$\mathbf{r}$	:	*An operating room.* $r \in \{1..R\}$
$\boldsymbol{S}$	:	*The number of surgeons.*
$\mathbf{s}$	:	*A Surgeon.* $s \in \{1..S\}$
Ω_s	:	*The set of operations allocated to the surgeon* s.
Ω_b	:	*The set of operations that takes place early in the day.*
Ω_e	:	*The set of operations that takes place at the end of the day.*
d(o)	:	*The duration of operation o (in number of time slots).*
ES_o	:	*The earliest start of operation o (time slots).*
LS_o	:	*The latest start of operation o (time slots).*
K^{ρ}	:	*The set of the renewable resources.*
K^{υ}	:	*The set of the non-renewable resources.*
K	:	*A resource.* $k \in \{K^{\upsilon} \cup K^{\rho}\}$
m_{ok}^{ρ}	:	*The quantity of the renewable resource k required by operation o.*
m_{ok}^{υ}	:	*The quantity of the non-renewable resource k required by operation o.*
M_{kt}^{ρ}	:	*The quantity of the renewable resource k, required by operation o at moment t.*
M_{k}^{υ}	:	*The quantity of the non-renewable resource k, for the day.*
M_{st}^{S}	:	*The availability of the surgeon s at time t.* $M_{st}^{S} \in \{0,1\}$

Operations are defined by a set of characteristics. We consider that operations have beforehand predicted duration *d(o)*. Furthermore, for the comfort and insurance of the patient, we consider that operations can start as soon as possible (ES_o) and not later than (LS_o) hours of beginning (in time slots).

A set of resources also describes operations. Legally, a surgical operation requires at least one surgeon, one anesthetist, and two nurses. The material resources can be renewable (K^{ρ}) or non-renewable (K^{υ}) for example, sterile medical trays, which constitute the basic equipment of operations and require a long treatment of sterilization between two purposes, are non-renewable resources along the day. Human resources are typically renewable resources (K^{ρ}).

In order to write constraints concerning the recovery room some new notations are introduced.

B	:	*The number of recovery beds.*
b	:	*A recovery bed.* $b \in \{1..B\}$
db(o)	:	*The duration of recovery of operation o (in number of time slots).*

2.2 Hypotheses

A list of hypotheses has to be enumerated in order to fulfill the description of our framework:

- Human and material resources are available in a limited number in the operation rooms;
- Human and all material resources except recovery beds are always available whenever needed in the recovery room;
- The restrained capacity of recovery beds is taken into account. It is noteworthy that, in practice, when no bed is free at the end of the surgery, the patient stays in the operating room until the release of a bed. The patient can also be transferred into his/her hospitalization bed if he/she wakes up before one bed is free in the recovery room. In our case and thanks to several above-mentioned constraints, we consider that the patient does not wake up in the operating room;
- No surgeon can operate on more than one patient at the same time; similarly, no recovery bed can be occupied by more than one patient at the same time;
- All the scheduled patients are ready for their surgery at a given day;
- The induction time for each operation and the post-operation clean-up time before leaving the operating room are included in the operating time;
- The recovery beds in the SSPI are identical, that is the patient can be transferred towards any recovery available bed;
- Emergency cases are not taken into consideration;
- Once a surgical case gets started in an operating room, it cannot be interrupted, i.e. there is no pre-emption;
- The time needed to clean the operating rooms (setup) and to rearrange the recovery beds are not taken into consideration. Let us specify here that the setup times are independent of the operating sequence. However, a priority will be given to a set of operations as previously mentioned. For example, those which are particularly contaminating such as iatrogenic infection will be scheduled in end-of-day;
- We take into account material resources like sterile medical trays, which constitute the basic material of

the surgeries and require a long treatment of sterilization between two uses.

We are now going to mathematically formulate the daily scheduling problem of the operating theatre in mixed integer programming (first model).

3. FIRST MODEL

The daily scheduling problem of the operating theatre, described in section 2, can be mathematically formulated by a mixed integer program. This model was originally written by Roland *et al.* (2009) and adapted to take into account additional aspects in order to compare the two models.

The model defined by Roland *et al.* (2009) is aiming to reduce the costs of weekly timetables reducing possible extra-time at the end of each and the costs induced by the opening of a new operating room during a day. In order to compare our two models, we do not take into account this planning aspect and reduce the time horizon to one day in order to focus on the daily scheduling problem. Here, we focus on reducing the makespan of the operating theatre (which is in fact the completion time of the latest wakening in the recovery room).

The mathematical model introduced by Roland *et al.* does not take into account the second stage of the operating theatre, named recovery room. We incorporate this stage in the first model in order to consider the whole operating theatre (see added constraints (11)(12)(13)(14)) to be able to provide results similar to those of the second model.

As we are in the context of a block scheduling, and not in an open-scheduling the set of constraints (10) of the Roland *et al.* model has been modified. Each surgeon expresses his availability in a matrix called $M^S(s,t)$. The example in Table 1 shows that operations can be affected to surgeon 3 between 8.45 hr. and 10.00 hr.

	8 hr.	15	30	45	9 hr.	15	30	45	10 hr.
T	1	2	3	4	5	6	7	8	9
Surg.#1	0	0	0	0	0	1	1	1	1
Surg.#2	1	1	1	1	1	0	0	0	0
Surg.#3	0	0	0	1	1	1	1	1	0

Table 1: Example of availability for surgeons

As previously mentioned, one set containing all the operations that have to be carry out earlier during the day", Ω_b, is created. Another set Ω_e, contains all the operations that have to be carry out at the end of the day. In addition, we can describe some earliest/latest start for each operation when needed (see added constraints (4)(5)(6)).

We define the x_{ort} binary variables, dedicated to the operations, which take the value 1 only when operation *o* starts in operating room *r* at time slot *t* (see figure 1). We define the y_{obt} binary variables, which are devoted to the recovery of the patient at the end of operation o when he/she begins to occupy the recovery bed *b* at time *t*.

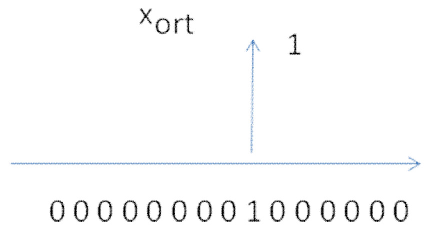

Figure 1: binary variable for model 1, and its vector

Therefore, the formulation of the model is now written as following:

Minimize $C_{\max}$ (1a)

s.t.

$$C_{\max} = \sum_{t=1}^{Min(t+d(o)-1,T)} \sum_{r=1}^{R} (t+d(o)).x_{ort}\ , \forall o \in \Omega \quad (1b)$$

$$\sum_{r=1}^{R}\sum_{t=1}^{T} x_{ort} = 1, \qquad \forall o \in \Omega \quad (2)$$

$$\sum_{o=1}^{O} \sum_{\tau=t}^{Min(t+d(o)-1,T)} x_{or\tau} \le 1, \qquad \forall r \in \{1...R\}, \forall t \in \{1...T\} \quad (3)$$

$$ES_o \le \sum_{r=1}^{R} \sum_{t=1}^{T} t.x_{ort} \le LS_o, \qquad \forall o \in \Omega \quad (4)$$

$$\sum_{t=1}^{T}\sum_{r=1}^{R} t.x_{o_1rt} \le \sum_{t=1}^{T}\sum_{r=1}^{R} t.x_{o_2rt}\ , \qquad \forall o_1 \in \Omega_b, \forall o_2 \in \overline{\Omega_b} \quad (5)$$

$$\sum_{t=1}^{T}\sum_{r=1}^{R} t.x_{o_1rt} \le \sum_{t=1}^{T}\sum_{r=1}^{R} t.x_{o_2rt}\ , \qquad \forall o_1 \in \overline{\Omega_e}, \forall o_2 \in \Omega_e \quad (6)$$

$$\sum_{o=1}^{O}\sum_{t=1}^{T} d(o).x_{ort} \le T, \qquad \forall r \in \{1...R\} \quad (7)$$

$$d(o).x_{ort} + d(o) - 1 \le T, \qquad \forall r \in \{1...R\}, \forall o \in \Omega, \forall t \in \{1..T\} \quad (7b)$$

$$\sum_{r=1}^{R}\sum_{o=1}^{O} m_{ok}^{\rho} \sum_{\tau=t}^{Min(t+d(o)-1,T)} x_{or\tau} \le M_{kt}^{\rho}, \qquad \forall t \in \{1..T\}, \forall k \in K^{\rho} \quad (8)$$

$$\sum_{r=1}^{R}\sum_{o=1}^{O} m_{ok}^{\upsilon} \sum_{t}^{T} x_{ort} \le M_{k}^{\upsilon}, \qquad \forall k \in \mathrm{K}^{\upsilon} \quad (9)$$

$$\sum_{r=1}^{R}\sum_{o\in\Omega_s}\sum_{\tau=t}^{Min(t+d(o)-1,T)} x_{or\tau} \le M^S(s,t),$$
$$\forall t\in\{1...T\}, \forall s\in\{1...S\} \quad (10)$$

$$\sum_{b=1}^{B}\sum_{t=1}^{T} y_{obt} = 1, \quad \forall o\in\Omega \quad (11)$$

$$\sum_{t=1}^{Min(t+db(o)-1,T)}\sum_{r=1}^{R}(t+d(o)).x_{ort} \le \sum_{t=1}^{T}\sum_{b=1}^{B} t.y_{obt}$$
$$\forall o\in\Omega \quad (12)$$

$$\sum_{o=1}^{O}\sum_{t=1}^{T} db(o).y_{obt} \le T, \quad \forall b\in\{1...B\} \quad (13)$$

$$\sum_{o=1}^{O}\sum_{b=1}^{B}\sum_{\tau=t}^{Min(t+db(o)-1,T)} y_{obt} \le B, \quad \forall t\in\{1..T\} \quad (14)$$

The constraints (2 and 11) ensure that an operation and recovery time take place once and only once. The set of constraints (3) ensures that the operations do not overlap. Each operation starts in the range bounded by the earliest/latest starting time thanks to constraints (4). Constraints (5 and 6) ensure both types of priorities. The first one, which allows the operation to pass early in the day, concerns for example diabetics, children or one day operations. The second, which forces some operations to take place at the end of the day, concerns infectious cases which contaminate the room and require a particular cleaning after the operation. The set of constraints (7 and 7b) bounds the limited capacity per room in number of available hours. The set of constraints (8) ensures the capacity limited by the renewable resources during the operation and the set of constraints (9) the capacity limited by the not renewable resources all along the day.

The set of constraints (10) verifies that the surgeon cannot be in two operating rooms by the same time. The surgeons cannot operate more than one patient by the same time. The surgeons operate in time slots attributed by a Master Plan of Allocation. Concretely, the availability of the surgeon at the moment *t* is given by an element of the matrix $M^S(s,t)$ which can be set to 0 for the related time slot "t" when the surgeon does not possess his(her) assigned availability in the Master Plan of Allocation. Conversely, if the MPA assigns a period of availability to the surgeon that includes the moment *t*, then the corresponding $M^S(s,t)$ element is set to 1.

The set of constraints (12) links the first stage to the second one. The recovery room has a capacity limited in the time (13) and in the number of recovery beds (14).

The following section presents our constraint programming model.

4. SECOND MODEL

We introduce below our daily scheduling problem model.
In order to express resources constraints more easily via logical operators (equality, difference, or/union, and/intersection, xor/mutual-exclusion…), we adopt a different implementation for the binary variable x_{ort} by comparison with the first model. Here x_{ort} takes the value 1 not only at the beginning of the operation but during all the duration of the operation. (see figure 2)

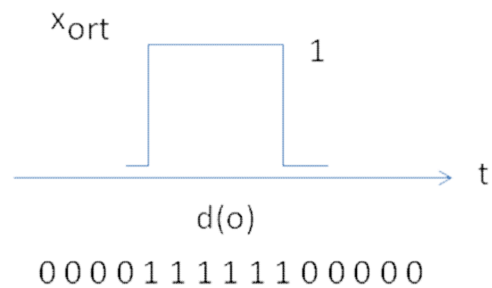

Figure 2: binary variable for an operation in model 2 and its vector (here d(o)=6)

So, within the same interval of time, the activity of each room is defined by means of R boolean variables. The operations have to be placed one by one, each one being described by the number of time-slots that they will take up in a room. Within a room *r*, the position of each operation is materialized by means of *d(o)* successive boolean variables showing the assignment of operation *o* in this room. We have to point out at this stage that only the boolean variables relative to the operation for the room in question and for the intervals of time occupied by the operation will be set to one and that everywhere else they will be set to zero. By this way, we induce a three-dimensional (O operations, T time slots and R rooms) boolean matrix. This matrix is represented by OTR (o, t, r).

Minimize C_{max} (15a)
s.t.

$$C_{max} = \frac{\left(\sum_{r=1}^{R}\sum_{t=1}^{T} t.OTR(o,t,r)\right) - \left(\frac{d(o).(d(o)-1)}{2}\right)}{d(o)} + d(o)$$
$$\forall o\in\{1..O\} \quad (15b)$$

$$\sum_{o=1}^{O} OTR(o,t,r) \le 1,$$
$$\forall r\in\{1..R\}, \forall t\in\{1..T\} \quad (16)$$

$$\sum_{r=1}^{R}\sum_{o\in\Omega_s} OTR(o,t,r) \le 1,$$
$$\forall s\in\{1..S\}, \forall t\in\{1..T\} \quad (17)$$

$$\sum_{r=1}^{R}\sum_{t=1}^{T} OTR(o,t,r) = d(o),$$

$$\forall o \in \{1..O\} \quad (18)$$

$$\sum_{r=1}^{R} \sum_{j=1}^{T-d(o)+1} \left\lfloor \frac{\sum_{i=j}^{j+d(o)-1} OTR(o,i,r)}{d(o)} \right\rfloor = 1,$$

$$\forall o \in \{1..O\} \quad (19)$$

$$ES(o) \le \frac{\left(\sum_{r=1}^{R} \sum_{t=1}^{T} t.OTR(o,t,r)\right) - \left(\frac{d(o).(d(o)-1)}{2}\right)}{d(o)} \le LS(o)$$

$$\forall o \in \{1..O\} \quad (20)$$

$$\frac{\left(\sum_{r=1}^{R} \sum_{t=1}^{T} t.OTR(o_1,t,r)\right) - \left(\frac{d(o_1).(d(o_1)-1)}{2}\right)}{d(o_1)} \le \frac{\left(\sum_{r=1}^{R} \sum_{t=1}^{T} t.OTR(o_2,t,r)\right) - \left(\frac{d(o_2).(d(o_2)-1)}{2}\right)}{d(o_2)},$$

$$\forall o_1 \in \Omega_b, \forall o_2 \in \overline{\Omega}_b \quad (21)$$

$$\frac{\left(\sum_{r=1}^{R} \sum_{t=1}^{T} t.OTR(o_1,t,r)\right) - \left(\frac{d(o_1).(d(o_1)-1)}{2}\right)}{d(o_1)} \le \frac{\left(\sum_{r=1}^{R} \sum_{t=1}^{T} t.OTR(o_2,t,r)\right) - \left(\frac{d(o_2).(d(o_2)-1)}{2}\right)}{d(o_2)},$$

$$\forall o_1 \in \overline{\Omega}_e, \forall o_2 \in \Omega_e \quad (22)$$

$$\sum_{r=1}^{R} \sum_{o=1}^{O} \frac{m_{ok}^{\rho}}{d(o)} \sum_{\tau=t}^{Min(t+d(o)-1,T)} OTR(o,\tau,r) \le M_{kt}^{\rho},$$

$$\forall t \in \{1..T\}, \forall k \in \mathrm{K}^{\rho} \quad (23)$$

$$\sum_{r=1}^{R} \sum_{o=1}^{O} \frac{m_{ok}^{\upsilon}}{d(o)} \sum_{t} OTR(o,t,r) \le M_{k}^{\upsilon},$$

$$\forall k \in \mathrm{K}^{\upsilon} \quad (24)$$

$$\sum_{r=1}^{R} \sum_{o \in \Omega_s} \frac{1}{d(o)} \sum_{\tau=t}^{Min(t+d(o)-1,T)} OTR(o,\tau,r) \le M_{st}^{S},$$

$$\forall t \in \{1...T\}, \forall s \in \{1...S\} \quad (25)$$

$$STR(s,t,r) = \sum_{o \in \Omega_s} OTR(o,t,r),$$

$$\forall s \in \{1..S\}, \forall t \in \{1..T\}, \forall r \in \{1..R\} \quad (26)$$

$$\sum_{r=1}^{R} STR(s,t,r) \le 1,$$

$$\forall t \in \{1..T\}, \forall s \in \{1..S\} \quad (27)$$

$$\sum_{o=1}^{O} OTB(o,t,b) \le 1,$$

$$\forall b \in \{1..B\}, \forall t \in \{1..T\} \quad (28)$$

$$\sum_{b=1}^{B} \sum_{t=1}^{T} OTB(o,t,b) = db(o),$$

$$\forall o \in \{1..O\} \quad (29)$$

$$\sum_{b=1}^{B} \sum_{j=1}^{T-db(o)+1} \left\lfloor \frac{\sum_{i=j}^{j+db(o)-1} OTB(o,i,b)}{db(o)} \right\rfloor = 1,$$

$$\forall o \in \{1..O\} \quad (30)$$

$$\frac{\left(\sum_{r=1}^{R} \sum_{t=1}^{T} t.OTR(o,t,r)\right) + \left(\frac{d(o).(d(o)-1)}{2}\right)}{d(o)} \le \frac{\left(\sum_{b=1}^{B} \sum_{t=1}^{T} t.OTB(o,t,b)\right) - \left(\frac{db(o).(db(o)-1)}{2}\right)}{db(o)},$$

$$\forall o \in \{1..O\} \quad (31)$$

The first set of constraints (16) expresses the fact that two operations cannot overlap at the same time in the same operating room. Furthermore, there is an exact matching between every operation and its surgeon. Thus, operations of each surgeon, selected amongst the set of surgeons S, are known in advance. The set of constraints (17) prevents each surgeon from being able to operate two operations at the same time in different operating rooms. In order to express the fact that every operation o has to take place on d(o) consecutive intervals of time, to set of constraints (18 and 19) have been introduced in the model. The first one imposes that the number of variables set to 1 is equal to d(o). The second imposes, thanks to the whole part of the sum balanced on an interval of time d(o), the continuity of the variables set to 1 in this interval. By this way, we express the fact that the successive "1" representing the operation is present only once (zero everywhere else). The set of constraints (4) of the first model is transcribed in the set of constraints (20) in this second model. A corrective factor allows us to locate the first 1 existing during *o* and to express comparable values to the previous model. Similarly, constraints (21 and 22) express the priority constraints of the operations included in the sets Ω_b and Ω_e. Constraints (23, 24 and 25) represent the constraints on the renewable and non-renewable resources. The difference lies in the fact that we have to balance by the opposite of the duration of every operation to come back to comparable values. In constraints (26 and 27) that are coming from (16, 17 and 19) and also in a redundant way, by means of STR matrix, it is possible to easily express the fact that a surgeon cannot operate at the same time in two different rooms via the constraints (20). Constraints (28, 29 and 30) express the conditions on the second stage, for instance the uniqueness of a patient in a recovery bed and the continuance of 1 on an interval of time d(o). The set of constraints (31) ensures the continuance between first and second stage; the left member indicating the last 1 representing the operation to the first stage and the right member representing the first 1 of the recovery of the patient in the second stage.

Thanks to the whole formulation of both models we are now able to compare these models and to estimate the most successful one.

5. EVALUATION AND OPTIMISATION

This section presents a comparison between both proposed models described in sections 3 and 4. These models have been implemented on a Pentium IV processor (3.2 GHz, 1.5 GB RAM, Operating System: Windows XP).

The first model, the mixed-integer programming, has been coded in Ampl language (Fourer, 2002) and solved by the solver Cplex 10.0 (Ilog, 1987). Cplex is a major product release that incorporates the latest enhancements in both solution speed and flexibility for mathematical programming.

The second model is based on the constraint programming method. It has been solved by the solver included in the java library Choco 2.1 (Rochart *et al.*, 2008), using JDK 1.6. This method is perfectly suited to the required descriptive power in order to accurately model the tackled problems. Built on an event-based propagation mechanism with backtrackable structures, constraint programming also offers the advantage of having different ways of functioning in the search of the tree. Either the path is relatively short and allows to quickly generate a feasible solution via a depth-first search algorithm, either contrarily a more sophisticated search (Branch-and-bound type) can be carried out in order to find an optimal solution, but at the price of a search time subject to become extremely important (Fages, 1996).

In order to evaluate the proposed methods in improving the practical arrangement of surgical cases in the operating theatre, real-life data of a Belgian University Hospital are used in this study. In this hospital, there are 9 surgical specialties: Stomatology, Gynaecology, Urology, Orthopedic surgery, ENT/Oto-rhino-laryngology, Ophthalmology, Pediatric surgery, Plastic surgery and Abdominal surgery. In practice, a variant of the block scheduling strategy is implemented, i.e. most of the time blocks are assigned to specific surgeons while some of them are assigned to specialties (e.g. Plastic surgery, Ophthalmology and Stomatology). In the latter case, any surgeon can book a case under the blocks reserved for his specialty (see constraints (10) and (25)). The operating theatre in this hospital is composed of six operating rooms and one recovery room composed of 10 places. Normally, all the operating rooms are open from 8:00 a.m. to 4:00 p.m. The recovery room opens simultaneously with operating rooms and remains open until the last patient goes out of the operating theatre.

An important issue is to quickly provide the operating theatre manager with a good solution, which satisfies all constraints.

In this study, the experiments are based on 6321 records from the operating theatre, which were collected in a Belgian hospital over a one-year period. The data mainly consists of date of surgery, induction time, the start time and end time of surgery, time of the patient's leaving operating room, corresponding surgeon and specialty for each surgical case and admittance reason together with some personal information (such as the patient's birthday, gender, etc.). The overtime hours are not considered because we have defined as fixed the time slots for a working day. However the overtime hours could be taken into account via a specific hourly cost, higher than the hourly cost used during the day.

In both presented models, the objective is to minimize the makespan, but other performance measures are also available and could as well be used in the cost/objective function.

5.1 First experiment

In this section, we consider a simplified example, focusing on one single operating day from our 6321 records in our database. The dataset for this day consists of 12 surgeries, 7 surgeons, 4 operating rooms, 2 renewable resources (anesthetist, nurse, one for each operation) corresponding to a real day in this hospital.

This aim of this "first experiment" is to compare the performances of the two resolution methods described in this paper regarding the above-mentioned simplified example.

In both cases, the same surgical cases made by the same surgeons using the same resources are considered and the duration of each time-slot is set to 15 minutes.

	Model 1 *Cplex*	Model 2 *Choco*
Cmax (time slots)	28	28
Time (s)	60,109	92,917

Table 2: Experimentation

Table 2 compares the obtained makespan (Cmax) and the required computational time to solve the problem using both models. Table 2 shows that the first model (based on mixed integer programming) is faster than the second one (based on constraint programming). We credit this performance to the Cplex solver, which is the outcome of several years in mathematical optimization research.

As the search mode of the PPC depends on the search strategy applied to the tree, it seems interesting to evaluate the impact of the order of the data entry on the results of our PPC model. Table 3 presents the results based on twelve permutations of the data entry.

	Model 2 *Choco* *time(s)*
permutation #1	103,86
permutation #2	44,94
permutation #3	80,50
permutation #4	38,89
permutation #5	19,94
permutation #6	10,89
permutation #7	93,59
permutation #8	3,70
permutation #9	3,66
permutation #10	3,66
permutation #11	3,77
permutation #12	3,67
Average	34,25
Std. Dev.	36,59

Table 3: Obtained run times for the set of permutations

Table 3 indicates that the computational time can be improved in average. However, the high value of the standard-deviation leads us to be careful as for the relevance of the average result. These detailed results show that the computational time is better than those presented in Table 2 on 9 times over 12. In 5 cases on 12, the obtained times are below 4 seconds. This experimentation demonstrates the high dependence of the PPC model on the order of the input data when running in first-feasible solution search backtrack mode.

5.2 Experiment in different configurations

This aim of this "second experiment" is to compare the performances of the two resolution methods described for other simplified examples that we have developed.

The dataset for considered days consists of 10-14 surgeries, 5-7 surgeons, 2-4 operating rooms, 2 renewable resources (anesthetist, nurse, one for each operation) corresponding to real days in this hospital. (see Table 4)

Both models were tested in different configurations. First of all, our examples were restrained with the constraints of priority of Ω_b and Ω_e sets. For datasets 2 and 3, we notice that Cmax has been obtained in a reasonable time for the modeling in Choco while Cplex required a long time to find the optimal solution. The same situation can be observed for datasets 4 and 5 where we test the availability constraints and constraints regarding the earliest start ant latest start.

In a close future we will test this with the entire activated constraints in order to even more restrain our models.

		Data #1	Data #2 *	Data #3 ²	Data #4 ²	Data #5 ³
	# room	4	2	2	2	3
	# surg.	7	5	5	6	6
	# oper.	12	13	10	12	14
Choco	Cmax	28	34	21	21	20
	time (s)	32,25	43,989	3,54	63,6	42
Cplex	Cmax	28	34	21	21	20
	time (s)	60,11	56822	7181	37816	25150

* with precedence constraints

² with availability constraints

³ with ES and LS constraints

Table 4: Experiments in different configurations

6. CONCLUSION AND PERSPECTIVES

With the aim of comparing objectively two models of the daily scheduling problem of an operating theatre, we described their advantages and disadvantages.

Our comparison shows that the first model:

1. Takes its advantage in the numerous existing methods of resolution to investigate the space of solutions.
2. Is obliged to be composed of linear equations to be solved using mixed integer programming.
3. Does not allow to integrate sophisticated constraints such as those encountered in the real life without complicating the model in such a way that it becomes difficult to build, to generalize and to solve.

The advantages and disadvantages of the second model are the following:

1. It is based on a constraint programming language, which has a higher descriptive power than classical mixed integer programming languages (for instance through specific additional logical operators); moreover it allows non-linear equations.
2. It allows to take into account a large number of constraints because the solver takes its advantage of the parceling of the search space,
3. However, it is in the majority faster than the first model.

The presented results experimentally show that the second model is sufficient to find good solutions. However, when running in depth-first search mode, it also needs a correct parameterization of the search method, depending on the provided data (it depends as well on the order of the input data).

In the future, we plan to objectively compare these two models on the basis of different data sets and on the parameterization of the Choco solver.

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